

Basic Integration Rules & Procedures from Math 150

Basic Integration Rules ($a > 0$)

$$1. \int k f(u) du = k \int f(u) du$$

$$2. \int [f(u) \pm g(u)] du = \int f(u) du \pm \int g(u) du$$

$$3. \int du = u + C$$

$$4. \int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$5. \int \frac{du}{u} = \ln|u| + C$$

$$6. \int e^u du = e^u + C$$

$$7. \int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$$

$$8. \int \sin u du = -\cos u + C$$

$$9. \int \cos u du = \sin u + C$$

$$10. \int \tan u du = -\ln|\cos u| + C$$

$$11. \int \cot u du = \ln|\sin u| + C$$

$$12. \int \sec u du = \ln|\sec u + \tan u| + C$$

$$13. \int \csc u du = -\ln|\csc u + \cot u| + C$$

$$14. \int \sec^2 u du = \tan u + C$$

$$15. \int \csc^2 u du = -\cot u + C$$

$$16. \int \sec u \tan u du = \sec u + C$$

$$17. \int \csc u \cot u du = -\csc u + C$$

$$18. \int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$$

$$19. \int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$20. \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$$

Procedures for Fitting Integrands to Basic Rules

Technique

Expand (numerator).

Separate numerator.

Complete the square.

Divide improper rational function.

Add and subtract terms in numerator.

Use trigonometric identities.

Multiply and divide by Pythagorean conjugate.

Example

$$(1 + e^x)^2 = 1 + 2e^x + e^{2x}$$

$$\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$$

$$\frac{1}{\sqrt{2x-x^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$$

$$\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$$

$$\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1} = \frac{2x+2}{x^2+2x+1} - \frac{2}{(x+1)^2}$$

$$\cot^2 x = \csc^2 x - 1$$

$$\frac{1}{1+\sin x} = \left(\frac{1}{1+\sin x}\right)\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{1-\sin x}{1-\sin^2 x}$$

$$= \frac{1-\sin x}{\cos^2 x} = \sec^2 x - \frac{\sin x}{\cos^2 x}$$